Earthquakes

Charles Richter developed the Richter Scale for measuring the magnitude of earthquakes in 1935. Magnitudes are generally expressed as whole numbers or decimals given to one decimal place. Worldwide there are far more low magnitude than high magnitude earthquakes. The table below shows how the average annual frequency of earthquakes varies with magnitude. These figures are based on observations since 1900.

Description	**Magnitude**	**Average Annual Frequency**
Great Earthquakes	8 and higher	1
Major Earthquakes	7 – 7.9	18
Strong Earthquakes	6 – 6.9	120
Moderate Earthquakes	5 – 5.9	800
Light Earthquakes	4 – 4.9	6 200 (estimated)
Minor Earthquakes	3 – 3.9	49 000 (estimated)
Very Minor Earthquakes	2 – 2.9	approx 1 000 per day
	1 – 1.9	approx 8 000 per day

Data Source: US National Earthquake Information Center

A relationship has been discovered between the magnitude of earthquakes and their frequency of occurrence.

Complete the following table where N denotes the number of earthquakes per year with magnitude greater than or equal to M.

M	N	$\log_{10} N$
8		
7		
6		
5		
4		
3		
2		
1		

Draw a graph of $\log_{10} N$ against M.

Use your graph to find the relationship between N and M.

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Planetary Motion

The German astronomer Johann Kepler studied the relative motion of the planets and discovered a relationship between their orbital periods and their mean distances from the sun.

Assume that this relationship is a power law of the form

$$T = kR^n$$

where T is the period in days, R is the mean radius of the planet's path in millions of kilometres and k is a constant.

The following table gives values of R and T for the planets in our solar system.

Planet	**Mean Distance from Sun** **R (10^6 km)**	**Period of Revolution** **T (days)**
Mercury	58	88
Venus	108	225
Earth	150	365
Mars	228	687
Jupiter	778	4 329
Saturn	1 427	10 753
Uranus	2 870	30 660
Neptune	4 497	60 150
Pluto	5 907	90 670

Plot a graph of log T against log R using either logarithms to base 10 or natural logarithms.

Use your graph to find the power law relating the orbital period and the mean distance from the sun.

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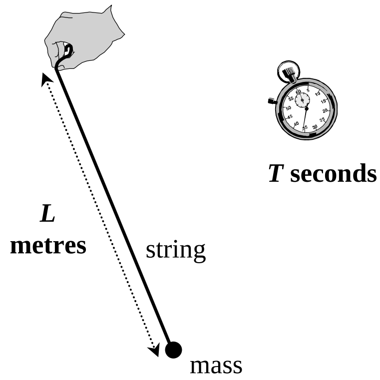
Log Graphs

Experiments

1 Tie an object to a piece of string.
Measure the average period of oscillation (T seconds) for different lengths of string (L metres).
Find the relationship between T and L.



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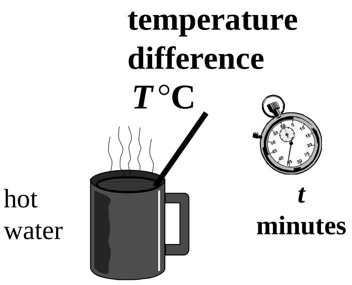
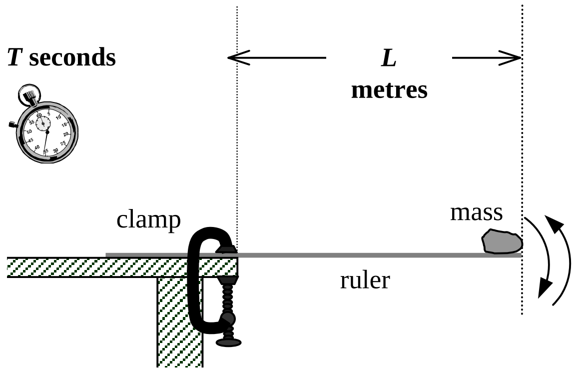


Pour some hot water into a mug or other container.
Measure the temperature difference between the water and the room $(T°C)$ at various times (t minutes) as the water cools.
Use a graph of ln T against t to find the relationship between T and t.

3 Attach an object to the end of a metre ruler.
Use a clamp to attach the ruler to the end of a table as shown.
Measure the average period of oscillation (T seconds) for different lengths (L metres).
Find the relationship between T and L.



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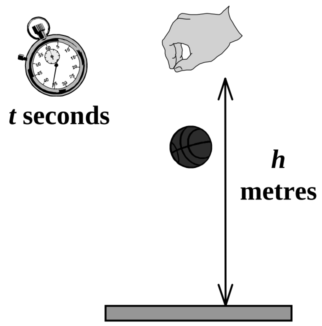
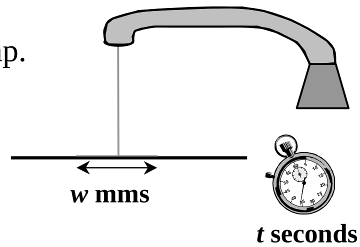


Drop a ball from various heights onto a hard surface.
Measure the time taken for the ball to stop bouncing (note that if it starts to roll it has stopped bouncing.
Find the relationship between the height (h metres) and the time taken (t seconds).

5 Adjust a tap until you obtain a light but steady stream of water.
Quickly insert a sheet of absorbent paper or material below the tap.
Hold it there for t seconds and then quickly remove it.
Measure the width of the water stain, w mm. immediately.
Repeat this for various times.
Find the relationship between w and t.



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Other Situations

1 Find the relationship between speed and braking distance using the data given in the Highway Code.

2 Find out what the population of Europe was at various times between the years 1700 and 2000. Deduce a relationship between population and year.

3 If a sum of money is invested in a savings account find the relationship between the amount in the savings account and the number of years for which the money has been saved. Choose a particular and annual rate of interest and assume that this remains constant. Also assume that that tax is not payable on the account and that the relationship may be represented by a continuous function.

4 Find the relationship between the value of a car and its age.
Either use data obtained from car trade magazines
or assume a particular initial value and (constant) annual rate of depreciation.

5 Find the relationship between the number of bacteria in a culture and the time if the number of bacteria doubles every 30 minutes. Assume an initial value for the size of the population.

6 The half-life of a radioactive isotope is the time taken for the mass of the radioactive isotope to reduce to half of the original mass. The table below gives the half-life of a number of radioactive isotopes. Choose one of these (or use another isotope if you wish) and assume a value for the original mass. Find the relationship between the mass and time.

Radioactive Isotope	**Half-Life**
Barium (Ba120)	32 seconds
Caesium (Cs137)	30 years
Carbon (C14)	5730 years
Cobalt (Co57)	270 days
Iodine (I131)	8 days
Plutonium (Pu239)	24 000 years
Potassium (K40)	1.28 billion years
Radon (Rn 99)	2.4 hours
Rhodium (Rh 99)	16 days
Thorium (Th 232)	14 billion years

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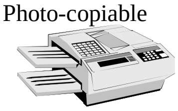

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Teacher Notes

Unit Advanced Level, Working with algebraic and graphical techniques

Notes

Pages 1 and 2 give examples that can be used to introduce log graphs. The calculations and graphs can be produced by hand or using a spreadsheet. (Using both of these methods would provide a check.) Answers and graphs that can be copied onto overhead projector transparencies for class discussion are given below.

Ideally students should use work related to their other studies and interests when producing work for their FSMU Coursework Portfolios. In some cases this may not be possible. Page 3 gives ideas for simple experiments and Page 4 gives suggestions of other real situations that could be used by students to give suitable data for log graphs. Use of these brief outlines would allow more able students to work independently, but it is likely that less able students will need much more help in collecting data and using it to derive the relationship required.

Earthquakes

The completed table is:

M	N	$\mathrm{Log}_{10} N$
8	1	0
7	19	1.2787536
6	139	2.1430148
5	939	2.9726656
4	7139	3.8536374
3	56139	4.7492647
2	421139	5.6244255
1	3341139	6.5238945

A graph of $\log_{10} N$ against M, produced using an Excel spreadsheet, is given on page 6.

The equation of the line of best fit is $\log_{10} N = 7.5 - 0.9M$ (to 1 dp)**.**

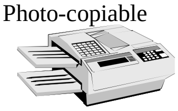
The relationship $\log_{10} N = a - bM$ is called the Gutenberg-Richter formula.

This relationship has been found to apply to particular regions as well as to the world as a whole. The constant a reflects the absolute level of seismicity of the region and the constant b has been found to be consistently close to the value 1. For the region of the UK from $10°W$ to $2°E$ and from $49°N$ to $59°N$ the Gutenberg-Richter formula has been found to be $\log_{10} N = 3.82 - 1.03M$

Planetary Motion

The graph of T against R given on page 7 can be used to show the shape often associated with power laws. A graph of $\log_{10} T$ against $\log_{10} R$, produced using an Excel spreadsheet, is given on page 8.

The trendline gives the equation $\log_{10} T = 1.5\log_{10} R - 0.7$ (to 1 dp) which can be rearranged to give Kepler's Third Law in the form $T = 0.2R^{1.5}$.



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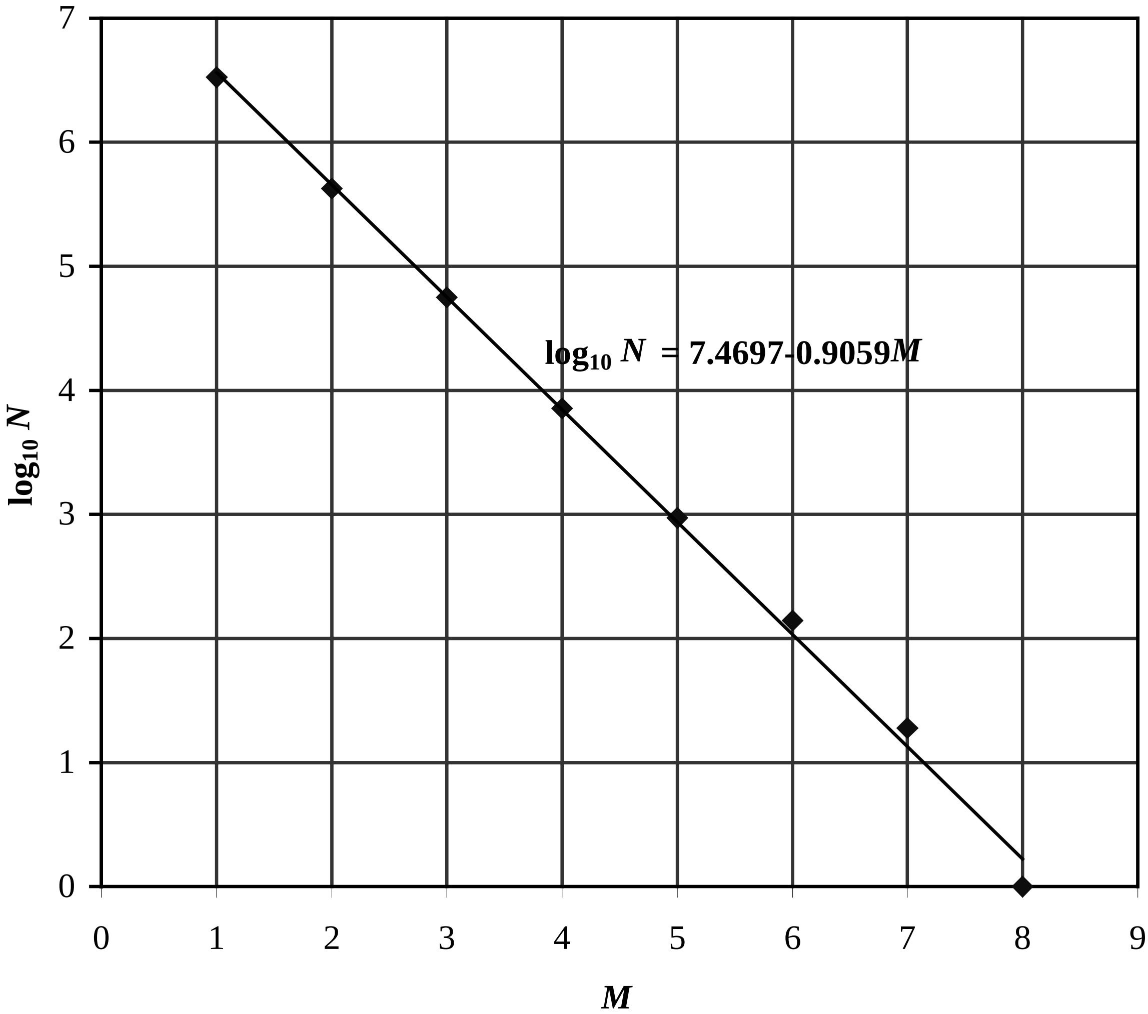
Graph of $\log_{10} N$ against M



$$\log_{10} N = 7.5 - 0.9M$$

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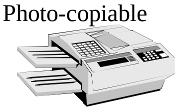

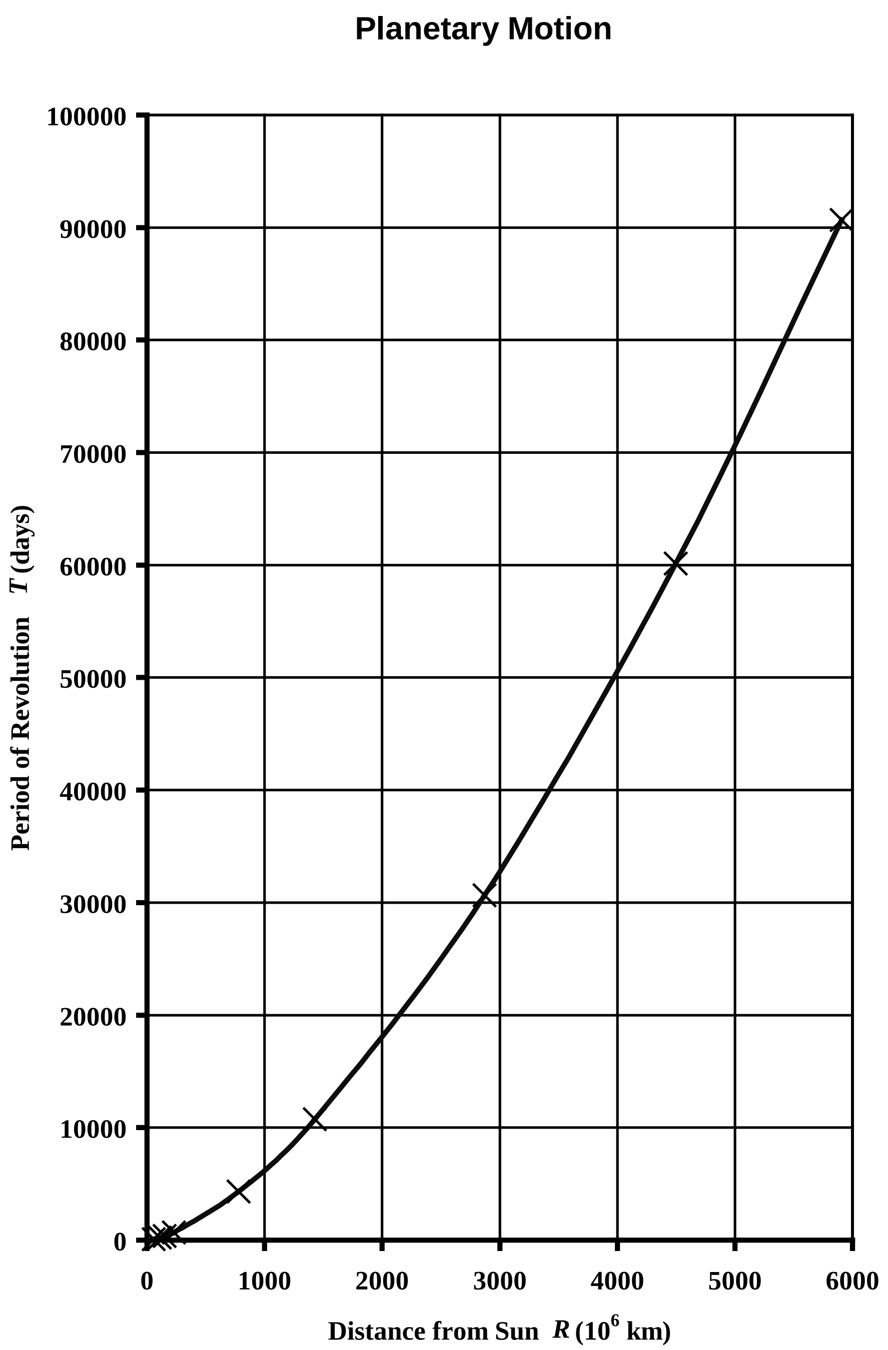

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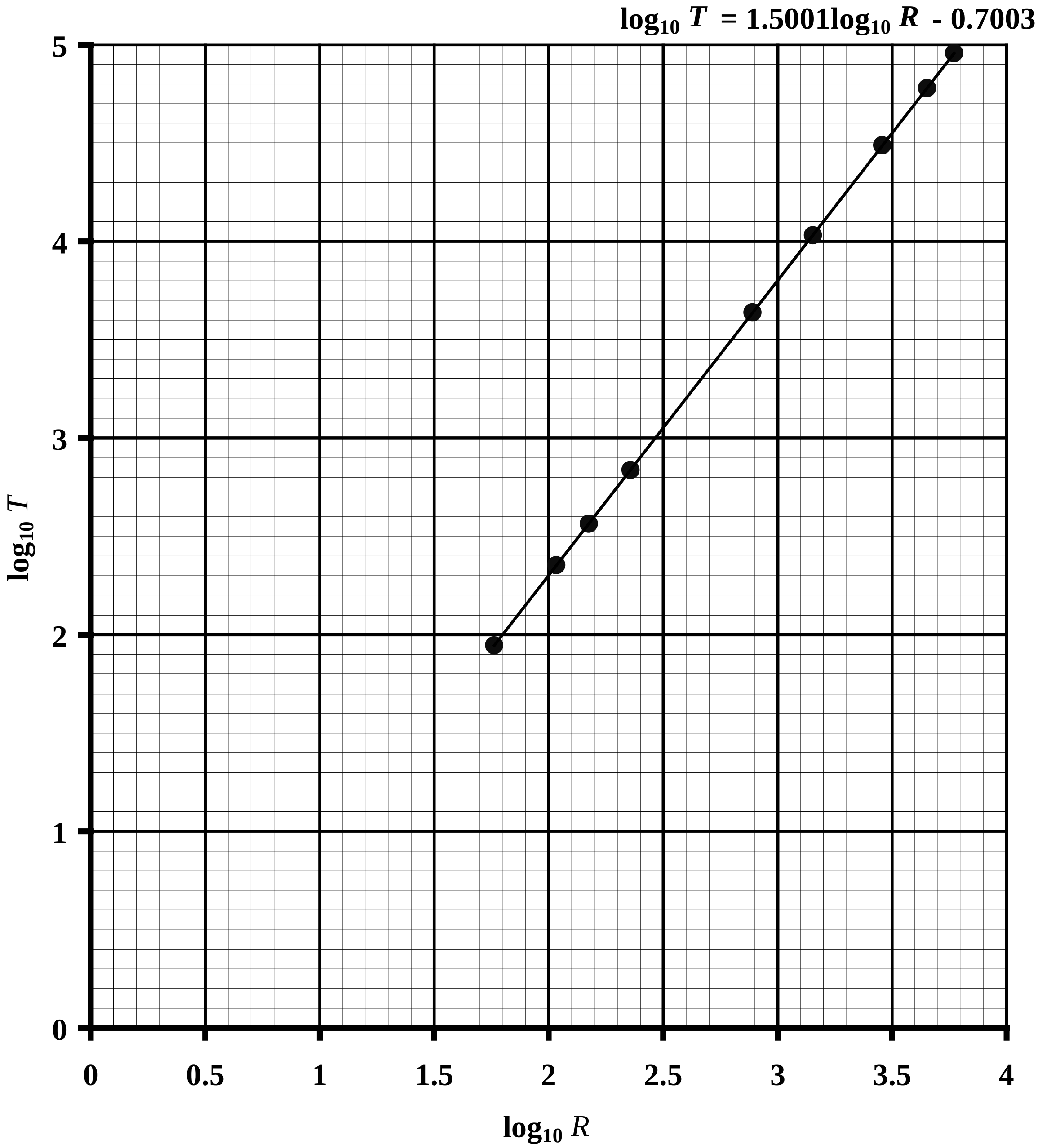
Graph of $\log_{10} T$ against $\log_{10} R$



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